

# An Examination of Force Ratios

A Monograph

by

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## Abstract

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The US Army is currently undergoing a transition from focusing on counter insurgency operations to large scale combat operations. As it undergoes this transition, the organization should reflect on its current doctrine and the use of heuristics such as force ratios. Therefore, the primary research question asks whether force ratios and quantitative models are valid tools for commanders and planners going forward. The underlying thesis of this study argues that force ratios are invalid and their continued use may develop unwanted mental constraints. By understanding the origins of force ratios and their evolutions, this study identifies a complete lack of consensus about the applicability of force ratios at various levels of war as well as challenges with common planning tools often associated with force ratios.

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## Acronyms

ADP	Army Doctrine Publication
ARVN	Army of the Republic of Vietnam
CHASE	Combat History Analysis Study Effort
COFM	Correlation of Forces and Means
HERO	Historical Evaluation and Research Organization
LSCO	Large-Scale Combat Operations
MACV	Military Assistance Command - Vietnam
NATO	North Atlantic Treaty Organization
QJM	Quantified Judgment Model
TAM	Theater Analysis Model
TNDM	Tactical Numerical Deterministic Model
TRADOC	Training and Doctrine Command
USSR	Union of Soviet Socialist Republics

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## Introduction

[T]o accept superiority of numbers as the one and only rule, and to reduce the whole secret of the art of war to the formula of numerical superiority at a certain time in a certain place was an oversimplification that would not have stood up for a moment against the realities of life.

— Carl von Clausewitz, *On War*

Current senior leaders in the US Army were young, impressionable company grade officers during the height of the Cold War. During this period, many individuals gravitated towards the scientific study of warfare as a means of justifying both the strategy and tactics upon which the North Atlantic Treaty Organization's (NATO) defense of Western Europe was based. Now in positions to implement change within the organization, many US Army leaders see the pivot from counterinsurgency operations to large-scale combat operations as one similar to the environment in which they led platoons and companies. As part of this transition, there is a growing impetus to return to the science of warfare, epitomized by reliance on heuristics such as force ratios to aid leaders in wargaming and decision making. The utilization of heuristics may be treacherous for military planners, particularly considering the fact that force ratios have not been consistently accurate when assessing historical cases.

Writing about how the human brain transitions between its intuitive functionality and its more deliberative and logical functionality, Daniel Kahneman highlighted the role played by heuristics. Kahneman described what he calls a simplifying heuristic as a rule of thumb that identifies a resemblance between a current situation and one already encountered to make a difficult judgment. He cautioned that the employment of resemblance through this heuristic might cause biases that lead to errors in predictions. Kahneman expanded on his premise that utilizing heuristics may result in erroneous predictions by asserting humans are “prone to exaggerate the consistency and coherence of what we see.”<sup>1</sup> As a result, humans tend to subconsciously identify

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<sup>1</sup> Daniel Kahneman, *Thinking Fast and Slow* (New York, NY: Farrar, Straus and Giroux, 2011), 7, 114.

causal connections between events, even when the connection is spurious.<sup>2</sup> One method of dealing with the simplification heuristic’s inherent risk is to track it from its creation and intended utilization, then account for evolutions in its use that may have occurred.

The US Army is a large organization that faces a myriad of internal and external challenges. Budget constraints in particular often have limited forces, driving planners to develop means of overcoming a numerically superior adversary. US military commanders have consistently struggled with the challenge of fighting a numerically superior enemy and winning. Numerous tools, tricks, and rules of thumb assist commanders and military planners seeking to simplify the planning process. Attempting to defeat a numerically superior enemy, US commanders mass combat power locally against perceived vulnerabilities and take advantage of temporary strength in both quantity and quality. One particularly common heuristic is the utilization of force ratios, which express a numerical advantage deemed necessary to prevail in a localized area over the enemy. The most common force ratio is the 3:1 rule, stipulating that success when attacking a prepared defensive position requires an offensive force with three times more troops than the defenders. A complete table of “Historical Minimum Planning Ratios” contained in current US Army doctrine is provided below.

**Table 1. Historical Minimum Planning Ratios**

<i>Friendly Mission</i>	<i>Position</i>	<i>Friendly : Enemy</i>
Delay		1:6
Defend	Prepared or fortified	1:3
Defend	Hasty	1:2.5
Attack	Prepared or fortified	3:1
Attack	Hasty	2.5:1
Counterattack	Flank	1:1

Source: US Department of the Army, Army Field Manual (FM) 6-0, *Commander and Staff Organization and Operations* (Washington, DC: Government Printing Office, 2016), Table 9-2.

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<sup>2</sup> Kahneman, *Thinking Fast and Slow*, 110.

Force ratios have endured in US Army doctrine as a simple, deterministic model based on firepower for predicting the outcome of a battle.<sup>3</sup> Meanwhile, the search for improved quantitative models has occurred throughout history. More rigorous quantitative approaches have sought to expand beyond measuring the relative firepower of forces first conducted by Fredrick Lanchester. Models such as the Quantified Judgement Model (QJM) and Correlation of Forces and Means (COFM) attempt to account for subjective factors of battles such as surprise, maneuver, and protection. As Kahneman pointed out, if an individual utilizes a heuristic, they assume risk. Intimate knowledge of the factors and context in which the heuristic was developed is the only way to mitigate that risk if it is to remain a useful tool for military planning scenarios. However, no doctrinal or academic discussion currently describes the context of force ratio development or implications of their use in planning. The nature of the inputs required for models such as the QJM or COFM mean that they are backwards looking, require numerous inputs, effort, and time to develop which limits their effectiveness to operational planners. Furthermore, COFMs in particular present the results of their quantitative comparison in the form of a force ratio. Therefore, COFM's utility is further reduced by the fact that force ratios have not withstood historical analysis.

Today, the predominant form of combat experience in the US Army consists of counter-insurgency operations in Iraq and Afghanistan. Fighting these diffuse and technologically limited enemies resulted in a degradation of planning skills for large-scale combat operations (LSCO) - particularly, the ability to assess forces required to accomplish missions. The primary question this study seeks to answer is whether force ratios found in US Army doctrine and quantitative models such as COFMs are applicable to future LSCO. Examining how the utilization of force ratios evolved in US Army doctrine may assist military planners in understanding the implication

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<sup>3</sup> Kevin Smith, "The Calculus of War: The Role and Use of Quantitative Decision Aids at the Tactical Level of War" (master's thesis, US Army Command and General Staff College, 1993), 72.

of utilizing them – and prevent their misuse. Moreover, appreciating the genesis and evolution of force ratios will demonstrate that the US Army should consider eliminating or modifying their future employment. For example, if force ratios were based on a purely linear, conventional battle, consisting only of the land domain, what are the implications of their use in modern warfare across all domains? Whether force ratios have been updated to account for the changing character of war is equally concerning. This analysis attempts to delineate the contextual parallels between past cases and those that describe the future environment of large-scale combat operations envisioned by the US Army.

This study is organized into five major sections, including this introduction. The second section will examine the origins of force ratios, focusing on the work and lasting impact of Frederick Lanchester. The third section of this study traces the early and varied usage of force ratios. It includes a brief look at the use of force ratios during the writing of the Victory Plan for World War II, their use as a training tool during peacetime, and their application in the employment of forces during Vietnam. This section highlights a lack of universal application of force ratios, particularly the employment of force ratios at various levels of war for different purposes. Section four presents the efforts undertaken to operationalize force ratios. It includes a brief history of the use of operations research and the search for quantitative analysis as it pertained to warfare. The section then focuses on the Army's transformation after Vietnam and the debate that occurred in the pages of *International Security* over how to best defend NATO operationally against a numerically superior enemy. Additionally, section four includes a discussion on the current incorporation of force ratios in Army Doctrine and the continued research efforts in the field of quantitative analysis as it pertains to warfare. The monograph concludes by arguing that heuristics lack the validity that military planners may unknowingly place on them. The subsequent level of risk that is passed on to military commanders through these assumptions is too high as the Army transitions its emphasis to Large Scale Combat Operations.

## Origins of Force Ratios

A force ratio is a comparison of the forces applied between two opposing sides. Force ratios emerged in the nineteenth century as a means of assessing the requisite amount of forces necessary to defend against an attacking force.<sup>4</sup> Frederick Lanchester was among the first to attempt to go beyond the general rule by using quantitative analysis to prove the force ratio's efficacy. Thus, to facilitate greater appreciation for the debate over the applicability of force ratios to warfare, this research starts with Lanchester. Understanding how force ratios developed unlocks the assumptions resident in their modern application, and probable limitations for future utilization. Moreover, despite Lanchester's work and subsequent quantitative approaches to understand warfare, there has yet to be an agreed upon approach; instead, these efforts have reinforced the use of force ratios as a heuristic.

Frederick Lanchester is one of the earliest known contributors to the field of quantitative analysis of warfare. In 1916, researching the utility of aircraft in warfare, Frederick W. Lanchester developed a differential equation that he referred to as the "N-Square Law." Lanchester was an aeronautical engineer who studied and published professional journals and books focused on aerodynamics, economics, industrial problems, and military strategy. Lanchester became widely known for his efforts to describe the impact that aircraft would have on warfare quantitatively. He believed that the capability of aircraft to efficiently concentrate fire in battle could offset the inefficiency of land armies.<sup>5</sup> Lanchester believed that airplanes could more efficiently accomplish an operation than land forces and that their efficiency would

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<sup>4</sup> Trevor N. Dupuy, *Understanding War: History and Theory of Combat* (New York, NY: Paragon House Publishers, 1987), 31. Dupuy claimed that President Lincoln's letter to Major General Halleck in 1863 in which Lincoln established that the ratio of forces defending Richmond could also be applied to those required to defend Washington DC. James E. Edmonds, *Military Operations: France and Belgium, 1917*, vol. 2 (London: The Imperial War Museum, 1948), 386. Edmond's identified German doctrine during the Franco-Prussian War of 1870 as espousing a requirement of 3:1 force ratio to defeat a defender of equal training, equipping, and morale.

<sup>5</sup> J. R. Newman, "Commentary on Frederick William Lanchester," in *The World of Mathematics*, vol. 4, ed. J. R. Newman (London: George Allen and Unwin Ltd., 1956), 2136.

ultimately determine the outcome of future battles.<sup>6</sup> With this background in mind, Lanchester developed an equation to measure the impact of the concentration of firepower. Lanchester believed that the principle of concentration was not just the strategic ability of a nation to divert the whole of its resources towards a common objective, but that it also applied to the tactical level as well. At the tactical level, Lanchester believed that the principle of concentration was of a purely scientific character.<sup>7</sup> His work was the first effort to quantitatively assign a modifying value to forces based on their efficiency or technological advantage. Previously, force ratios relied purely on counting the number of forces available as equals.

Lanchester scientifically expressed this idea with his N-Square Law. Published in his 1916 book *Aircraft in Warfare*, it sought to compare the relative strength of opposing armies and develop a reasonable prediction of the outcome of an engagement.<sup>8</sup> Lanchester concluded from his work that the defender retains a *tactical* advantage, requiring an attacker to possess a locally confined numerically superior force; but that *strategically*, the advantage remains with the attacking force because of the ability to select the point of concentration where they may ensure a local numerical superiority.<sup>9</sup> Although Lanchester utilized the term *strategic* because it was well known at the time, his use of the term aptly describes what the US Army has referred to as *operational* since it was incorporated into doctrine in 1982 to describe employing military resources to attain strategic goals.<sup>10</sup>

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<sup>6</sup> Frederick W. Lanchester, *Aircraft in Warfare, the Dawn of the Fourth Arm* (London: Constable and Company Ltd., 1916), xiii.

<sup>7</sup> *Ibid.*, 39.

<sup>8</sup> Newman, "Commentary on Frederick William Lanchester," 2136.

<sup>9</sup> Lanchester, *Aircraft in Warfare, the Dawn of the Fourth Arm*, 126-127.

<sup>10</sup> Richard M. Swain, "Filling the Void: The Operational Art and the U.S. Army," in *Operational Art: Developments in the Theories of War*, ed. B.J.C. McKercher and Michael Hennessy (Westport, CT: Praeger, 1996), 159-160.

Lanchester originally sought to determine what size force must be concentrated to overcome the defender's tactical advantage.<sup>11</sup> Furthermore, he emphasized material capabilities, which enabled him to focus on the scientific character of weaponry.<sup>12</sup> Lanchester argued that before the industrial revolution and modern weaponry, despite numbers on the field, generally speaking, war was man for man. He surmised that even if one side had a numerical advantage, the number of men that could actually wield their weapons against the enemy at a particular moment was roughly the same for both sides given an unbroken firing line and open terrain.<sup>13</sup> Lanchester acknowledged the impact that magazine fed rifles, machine guns, and artillery had on the battlefield and that they fundamentally altered the one-for-one exchange of ancient battle, which informed his development of the equation.

According to the N-Square law, the fighting strength of a force is proportional to the square of its numerical strength, multiplied by the fighting value of the individual units.<sup>14</sup> The N-Square is thus represented with the equation  $N r^2 = M b^2$ . In the equation,  $b$  stands for the numerical quantity of blue forces and  $r$  represents the quantity of red or opposing forces.  $M$  and  $N$  are representative values of the efficiency of the fighting force. Utilizing this equation, Lanchester demonstrated that a blue force of 500, using magazine fed rifles, suffers 100 casualties when it attacks a numerically superior red force of 1,000 soldiers armed with breech-loading rifles. Further, the red force experiences 200 casualties.<sup>15</sup>

Although the equation depicts the attrition of forces relative to one another, it does not predict what levels would constitute success on the battlefield, and was not originally intended to

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<sup>11</sup> Dupuy, *Understanding War*, 19.

<sup>12</sup> Lanchester, "Mathematics in Warfare," 2138-2139.

<sup>13</sup> *Ibid.*, 2139.

<sup>14</sup> *Ibid.*, 2145.

<sup>15</sup> Lanchester, *Aircraft in Warfare, the Dawn of the Fourth Arm*, 47-48.

deal with force attrition.<sup>16</sup> More broadly, while Lanchester believed the quantitative analysis of forces and a utilization of mathematical theory to predict outcomes of conflicts was utilitarian, he also understood the fact that you could not mathematically account for numerous unknown factors such as morale, leadership, and chance.<sup>17</sup>

Lanchester's N-Square Law therefore was a mathematical model that measured attrition at a point of concentration, a *tactical* application of force ratios. As a mathematical equation, the N-Square Law assumes perfect efficiency at the concentration point, an idea that events of World War I and subsequent wars readily disprove. James Schneider, a former professor of military theory at the US Army School of Advanced Military Studies, has pointed out that the N-Square Law represents Napoleonic and naval warfare. Battles of World War I and later break from the efficiency of Napoleonic or naval warfare theorized by Lanchester and are instead characterized by empty battlefields and distributed maneuver. The N-Square law has never been completely rejected and nearly all military simulations through at least the 1990s relied on some variation of the law, despite empirical evidence against its validity.<sup>18</sup>

In the 1960s and 1970s, operations research analysts returned to Lanchester's work to assess its utility based on the historical outcomes of engagements. Dr. Daniel Willard was among the first to conduct testing of Lanchester's work, utilizing data derived from a compilation of 1,500 engagements ranging from the Thirty Years War through the Russo-Japanese War. Willard concluded that the Lanchester Equations were not supported by historical data.<sup>19</sup> Dr. Janice Fain

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<sup>16</sup> Dupuy, *Understanding War*, 19.

<sup>17</sup> Lanchester, "Mathematics in Warfare," 2144.

<sup>18</sup> James J. Schneider, School of Advanced Military Studies Theoretical Paper No. Four., "Vulcan's Anvil: The American Civil War and the Foundations of Operational Art" (Fort Leavenworth, KS: SAMS/USACGSC, 1992), 2-5.

<sup>19</sup> Trevor N. Dupuy, *Numbers, Predictions, and War* (Fairfax, VA: HERO Books, 1985), 148-149. Daniel Willard, *Lanchester as a Force in History: An Analysis of Land Battles of the Years 1618-1905* (Bethesda, MD: Research Analysis Corporation, 1962), 3-4, 9. Willard utilized Kriegsllexicon database developed by Gaston Bodart in 1908. Willard took strength numbers, casualty numbers, as well as winners and losers from the Kriegsllexicon and transferred them onto punched cards to be read by a computer.



conducted a second study that utilized a narrower set of engagement data, focusing on World War II. Initially, she reached the same conclusion as Willard.<sup>20</sup> However, when Dr. Fain replaced the numerical values of opposing forces she gathered from the database with combat power potential ratios that accounted for the interactions of additional variables of combat beyond firepower, the results changed. Utilizing the combat power potential ratios, she concluded that the Lanchester Equations provided reliable casualty rates.<sup>21</sup>

Despite Dr. Fain's study, Lanchester's equation still lacked an ability to identify or link casualty rates to success on the battlefield. In other words, Dr. Fain concluded that casualties were not an indicator of success and that variables such as willpower, bravery, determination, and other courses of action available to the enemy all affect whether a force retires from the battlefield after 100 casualties or 1,000 casualties. Her study did identify a need for quantitative analysis to break from the trend of focusing solely on weapons systems and their effect and devote at least some effort to incorporating behavioral variables of combat.<sup>22</sup> Hence it launched the trend of incorporating variables beyond just firepower and weapons systems, as was later seen in the QJM, TNDM, and COFM.

### Early and Varied US Army Usages of Force Ratios

This section offers three cases in which the US Army employed force ratios, encompassing World War II, the 1950s peacetime Army, and Vietnam. The use of the concept varied among these cases, demonstrating a lack of consensus about their proper application, and particularly how to utilize them for different levels of war.

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<sup>20</sup> Dupuy, *Numbers, Predictions, and War*, 149. Dr. Fain utilized the HERO's 60-Engagement Data Base of WWII in Italy.

<sup>21</sup> Dupuy, *Numbers, Predictions, and War*, 149-150.

<sup>22</sup> Janice B. Fain, "The Lanchester Equations and Historical Warfare: An Analysis of Sixty World War II Land Engagements," *History, Numbers, and War* 1, no. 1 (Spring 1977): 43.

## World War II

The extant literature that specifically addresses the use of force ratios from the World War II period is limited, but military planners did emphasize the importance of numerical superiority. In 1941 the US War Department developed the Victory Plan under the guidance of Major Albert Wedemeyer. It was a plan for both the mobilization of forces and the employment of those forces in World War II that balanced domestic and military manpower priorities. In effect, the Victory Plan was a crucial component of the United States' guiding strategy for World War II.<sup>23</sup>

Developing the Victory Plan, planners initially focused on achieving an overall numerical superiority of 2 to 1 “normally considered necessary before undertaking offensive operations.”<sup>24</sup> However, Wedemeyer identified that there was no possibility of the Allied forces fielding the requisite 700 to 900 divisions required to achieve a 2:1 ratio over the German Army in Europe. Instead, Wedemeyer focused his plan on how, the United States, a numerically inferior nation could leverage “fighting machines and air forces” to achieve victory.<sup>25</sup> Thus, Wedemeyer abandoned a course of action focused on numerical strength in favor of superior technology, a path that would be repeated in the future.

Wedemeyer's requirement for a 2:1 superiority illustrates that force ratios were a consideration for military planning. Furthermore, Wedemeyer understood the rule to apply to the total means of the nation and its ability to field divisions in the theater. Wedemeyer's understanding of force ratios represents their utility at the *strategic* level of war. His application of force ratios does not appear to recognize Lanchester's work, as there is no mention as to the applicability of achieving numerical superiority at a localized point of concentration. Instead,

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<sup>23</sup> Charles Kirkpatrick, *An Unknown Future and a Doubtful Present: Writing the Victory Plan of 1941* (Washington, DC: US Army Center of Military History, 1992), 122.

<sup>24</sup> *Ibid.*, 82.

<sup>25</sup> *Ibid.*, 82-83.

Wedemeyer applies force ratios at the strategic level as a means of balancing numerical shortfalls with military capabilities instead of predicting the outcomes of individual battles.

## 1950s Doctrinal Training Aid

Following the Korean War, the US Army underwent budget and manpower reductions while retaining the requirement to maintain a high level of preparedness should war breakout between the US and the Soviet Union or China. As a result, the Army struggled with how to maintain a high quality, trained force manned by a peacetime pool of draftees.<sup>26</sup> To assist with standardizing training, the Army revised *Maneuver Control* in 1955 which served as a means of prescribing how to umpire tactical exercises between United States forces and those representing an aggressor. Umpires, using this manual as a base, enforced standardized, logical, and realistic results of wargames to allow commanders and staffs a demanding training environment.<sup>27</sup>

Army doctrine incorporated force ratios for the first time in *Maneuver Control*, a training manual published in 1955. The manual charged umpires with simulating an atmosphere of a battlefield by determining the results of contacts after considering the relative strength of each side and assessing casualties to both soldiers and equipment.<sup>28</sup> The manual dictated that a unit should only advance after it was determined that the element possessed a decisive superiority of fire, which it stipulated was seldom less than 2:1, and in general should be 3:1 or 4:1. In instances when the defender is behind cover and has clear fields of fire, the ratio increased to 5:1 or greater.<sup>29</sup>

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<sup>26</sup> Donald A. Carter, *The US Army Before Vietnam, 1953-1965* (Washington, DC: US Army Center of Military History, 2015), 7-11.

<sup>27</sup> US Department of the Army, *Field Manual (FM) 105-5, Maneuver Control* (Washington, DC: Government Printing Office, 1955), 3.

<sup>28</sup> *Ibid.*, 34.

<sup>29</sup> *Ibid.*, 58-59.

Like Lanchester's equations, this manual explicitly pointed out that only those units physically engaged in the fight count towards the firepower score.<sup>30</sup> Forces designated as the reserve or forces incapable of supporting the fight due to canalizing terrain or range limitations were not considered. The manual also provided umpires reference tables to calculate firepower scores based on weapons, unit size, and range of the engagement. The firepower score of a unit was multiplied when the unit used an attack by fire position and maneuvered while the defense was suppressed, or when the unit attacked the flank or rear of an opposing force.<sup>31</sup>

The 1955 version of *Maneuver Control* was a purely tactical application of force ratios. It sought to apply force ratios to training exercises in a manner which provided standardized outcomes to engagements. Umpires applied force ratios at the tactical level for a variety of purposes, including assessing overall success, number of casualties, and damage to equipment. Reflecting later on the strict adherence to force ratios by umpires of field training exercises, General Omar Bradley expressed his belief that the ratios had developed a mindset of tactical leaders that failed to account for the intangible aspects of warfare. Bradley cited numerous cases where units constrained their frame of reference by using force ratios, either by failing to seize the initiative or surrendering unnecessarily.<sup>32</sup>

Kahneman's cautions against using resemblance between a current situation and one already encountered to make a difficult judgment because it may lead to errors in predictions. As a result of interactions with umpires employing force ratios, Bradley asserted that commanders were making that very error during training exercises. With the Army's focus now transitioning back to LSCO, it must be cognizant of the danger present in relying on force ratios. Continuing to

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<sup>30</sup> US Army, *FM 105-5* (1955), 62.

<sup>31</sup> *Ibid.*, 59-69.

<sup>32</sup> Robert S. Cameron, *Mobility, Shock, and Firepower: The Emergence of the US Army's Armor Branch, 1917-1945* (Washington, DC: US Army Center of Military History, 2008), 389-390.

stress their application in training exercises may erroneously restrict commander's mindsets and ultimately affect their acceptable courses of action.

## Vietnam

As the Army contended with containing the conventional threat posed by the Soviet Union, US policymakers also employed force ratios in counterinsurgency operations in Vietnam. The US engagement in Vietnam began with the deployment of Special Forces advisors to the South Vietnamese Army in 1957, and gradually escalated to its peak of 365,000 troops in South Vietnam by 1969 with thousands more providing direct support to operations.<sup>33</sup> Prior to the commitment of combat troops in 1965, senior military planners struggled with how to convey force requirements. Military planners within Military Assistance Command Vietnam (MACV) relied on force ratios to communicate their requirements to the Joint Chiefs of Staff. MACV used a force ratio of 10 counterinsurgents for every 1 insurgent force or a ratio of 10:1. Utilizing this ratio, MACV illustrated how the commitment of a US battalion would improve the counterinsurgent ratio.<sup>34</sup>

However, MACV did not rely solely on a quantitative comparison of forces, they also incorporated a qualitative assessment. MACV assessed that the quality of a US Army battalion was equal to two Viet Cong battalions and that a Marine Corps battalion was equal to three Viet Cong battalions.<sup>35</sup> Analyzing the anticipated troop increase in 1965, MACV determined that “with the 13 Army and 4 Marine battalions, the ARVN thus would gain the equivalent of 38 of its own battalions.”<sup>36</sup> Despite the qualitative adjustment in equivalency, the troop ratio within Vietnam was 1.9:1 in favor of the Viet Cong. General Westmoreland determined it was not

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<sup>33</sup> Carter, *The US Army Before Vietnam, 1953-1965*, 48, 5-6.

<sup>34</sup> Graham A. Cosmas, *MACV: The Joint Command in the Years of Escalation, 1962-1967* (Washington, DC: US Army Center of Military History, 2006), 204.

<sup>35</sup> Ibid.

<sup>36</sup> Ibid.

feasible to attain a 10:1 ratio of forces in South Vietnam, despite the qualitative adjustment, and abandoned the ratio in favor of a conventional attack ratio of 3:1. He cited the combat power that artillery and air power brought to bear to justify a reduction in the necessary force ratio.<sup>37</sup>

Westmoreland's incorporation of air power and artillery represents the use of a heuristic since there is no evidence that it was quantified through an analysis of the actual contribution of those enablers. Furthermore, his use of force ratios to justify additional troop strength required in Vietnam was at the strategic level of war, and did not address their operational or tactical employment.

### Towards the Use of Ratios for Operational Planning

The Cold War between the United States and the Soviet Union expanded the quantitative analysis and correlation of forces between the two superpowers as the United States sought to gain an advantage. Conducting a comparative assessment of forces was a *strategic* mechanism that drove the size and characteristics of the military branches.<sup>38</sup> The use of force ratios now expanded, despite a lack of consensus among academics and practitioners on its proper application. This section highlights the work of operations research analysts, particularly those produced by the Historical Evaluation and Research Program (HERO), and how it contributed to the Army's transformation of the 1960s and 1970s. Finally, this section highlights the academic debate that occurred in the 1980s over the use of force ratios for operational planning.

### Operations Research

The Army originally entered the field of operations research during World War II as a means of making scientifically demonstrated recommendations for improvements within the Army. Operations research collected and analyzed historical data to make recommendations to

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<sup>37</sup> Cosmas, *MACV: The Joint Command in the Years of Escalation, 1962-1967*, 223.

<sup>38</sup> "Introduction: An Assessment of U.S. Military Power," The Heritage Foundation, October 4, 2018, accessed December 31, 2018, <https://www.heritage.org/military-strength/assessment-us-military-power>.

military decision makers about improving weapons and equipment, organizational structure, doctrine, and strategy. In 1962, Secretary of Defense Robert McNamara reorganized the Army along functional lines and consolidated various commands, producing the US Army Combat Development Command.<sup>39</sup> To meet growing demands for operational research, the US Army also began contracting some of its analysis to civilian organizations and think tanks.

One of these organizations was the HERO, founded in 1962 by Trevor N. Dupuy. It initiated and pioneered the field of simulation and modeling for military application in the 1960s, establishing Dupuy as a prominent figure in the operational research field by the 1970s. Dupuy was a United States Army Colonel (Retired), a historian, and an avid writer. He was also a member of the founding faculty for Harvard's Defense Studies Program. Under Dupuy's tutelage, HERO examined battles, after action reports, and case studies to discern what factors contributed to the outcomes of those battles. In 1992, Dupuy established the Dupuy Institute as a non-profit corporation to continue his examination of military history and the measurement of lethality on the battlefield.<sup>40</sup>

While leading HERO, Dupuy oversaw the Combat History Analysis Study Effort (CHASE) which sought to establish a computer-readable database of battles that could be used by operational researchers to analyze trends and interrelationships and to test hypotheses. Dupuy hoped that such analysis would provide a better understanding of what is referred to as combat variables such as forces, circumstances, and doctrine.<sup>41</sup> Both Dupuy and HERO acknowledged inherent limitations in such efforts. The most important limitation was that the strengths of opposing forces was (and even now remains) incomplete. Second, the database established by

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<sup>39</sup> Charles R. Shrader, *History of Operations Research in the United States Army*, vol. 1, 1942-62 (Washington DC: Government Printing Office, 2006), v.

<sup>40</sup> Susan Rich, "Trevor N. Dupuy," The Dupuy Institute, accessed September 27, 2018, <http://www.dupuyinstitute.org/tndupuy.htm>.

<sup>41</sup> Robert L. Helmbold and Ageel A. Khan, *Combat History Analysis Study Effort (CHASE): Progress Report for the Period August 1984-June 1985* (Bethesda, MD: US Army Concepts Analysis Agency, 1986), iii, v; Dupuy, *Understanding War*, 55-56.

HERO was limited to 601 battles that took place between 1600 and 1973, and not large enough to adequately serve as a representative sample.<sup>42</sup> Lastly, the databases only contain the historical outcomes and lack information pertaining to developments that occurred during the course of the engagement. Despite these limitations CHASE determined that force ratios are “an unsatisfactory and inadequate predictor of victory in battle.”<sup>43</sup>

Still, Dupuy endeavored to use quantitative analysis of historical battles as a means to predict the outcome of future engagements. Dupuy's work led him to create the Quantified Judgement Model (QJM). Dupuy believed that the QJM provided a basis for comparing the relative combat power of opposing forces and could account for the influence of variable factors resulting in a predicted outcome.<sup>44</sup> This model analyzed the numerical strengths of opposing sides in an engagement while also accounting for considerations of surprise, terrain features, defensive posture, and other circumstances of battle. The result was another quantitative approach to understanding the forces and circumstances required to win an engagement.<sup>45</sup>

Dupuy's efforts to quantitatively analyze warfare and provide a tool for prediction is in itself not an instance of force ratios. In fact, he criticized the use of force ratio because it did not indicate what was to be considered, whether it was numbers of units, soldiers, like-weapons, firepower, or other factors. He pointed out that it is not difficult to assemble a database of battles that justify force ratios and it is just as easy to compile a different database that disproves their

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<sup>42</sup> Helmbold and Khan, *Combat History Analysis Study Effort (CHASE): Progress Report for the Period August 1984-June 1985*, 1-2. Helmbold and Khan argue that to be a representative database of land combat battles, the database would have to be expanded to include sea and air battles, actions from the Korean, Malayan, Algerian, and Vietnamese Wars, more World War II battles before 1942, and should be expanded to include Asian, African, Mideast, and South American Wars.

<sup>43</sup> Helmbold and Khan, *Combat History Analysis Study Effort (CHASE): Progress Report for the Period August 1984-June 1985*, 1-6.

<sup>44</sup> Dupuy, *Understanding War*, 280.

<sup>45</sup> Trevor N. Dupuy, *The Evolution of Weapons and Warfare* (Fairfax, VA: HERO Books, 1984), 332-333.



utility.<sup>46</sup> Instead, his work expanded beyond a ratio of forces by applying information about forces within the context of their operational environment. Dupuy's QJM accounted for weapons effectiveness and quantity, mobility, and vulnerability. It also accounted for terrain, environmental factors, leadership, training, and logistics.<sup>47</sup> Though quantitative in application, deriving the quantities used in comparison still relied on a substantial amount of subjectivity. To mitigate this concern, users of QJM received with the database the values assigned to various factors to ensure transparency and consistency across their assessments.<sup>48</sup>

The QJM was developed as a computer program to conduct wargaming by replicating the effects of weapons and the external factors of their employment.<sup>49</sup> The QJM was validated internally by Dupuy's HERO in the 1970s against a selection of 200 hundred battles from World War II and the 1967 and 1973 Arab-Israeli battles.<sup>50</sup> Comparing the efficacy of wargaming models available for the Army in the 1990s, David Hogg recommended that the US Army adopt the QJM for wargaming at the Division and above echelons. Hogg credited the QJM with being a better model than the Theater Analysis Model (TAM) that served as the basis for the correlation of forces model utilized by the TRADOC Analysis Command.<sup>51</sup>

Despite the QJM's limited success and Hogg's assessment that it was most accurate, it still poses challenges for operational planners. The model is backwards looking, meaning that it is using historically provided inputs to assess historical outcomes. Hence it is not effective for future operations where obtaining required inputs would be clouded in uncertainty – particularly against an enemy force actively seeking to protect against the disclosure of information. As many

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<sup>46</sup> Dupuy, *Numbers and Military History*, 11-13.

<sup>47</sup> *Ibid.*, 52-53.

<sup>48</sup> *Ibid.*, 185-231.

<sup>49</sup> David Hogg, "Correlation of Forces: The Quest for a Standardized Model" (monograph, School of Advanced Military Studies, US Army Command and General Staff College, 1992), 19.

<sup>50</sup> Christopher Lawrence, *War by Numbers* (Lincoln, NE: Potomac Books, 2017), 300.

<sup>51</sup> David Hogg, "Correlation of Forces: The Quest for a Standardized Model," 40.

inputs are situationally specific, the need to update the inputs for each engagement creates a time burden that further reduces the QJM's utility for planners. Despite its ineffectiveness as a planning tool, backwards looking historical models such as this can provide accurate simulations and generate training aids for hypothetical scenarios of historical case studies.

## Army Transformation

In 1961 Robert McNamara, a World War II veteran and former president of Ford Motor Company, became the US Secretary of Defense. McNamara faced the problem that the US was in a long-term conflict with the Soviet Union, and its success clearly depended upon the nation's economic health.<sup>52</sup> Moreover, by the 1960s, the US no longer held a nuclear superiority over the USSR, and nuclear parity meant that the Department of Defense had to improve its technology and doctrine if it was to defend NATO.<sup>53</sup> McNamara was concerned with efficiency and ensuring that the nation got its money's worth from the defense allocations. He imposed modern business and budgetary practices on the armed forces.<sup>54</sup> McNamara replaced Eisenhower's doctrine of "massive retaliation" and introduced the strategy of "flexible response" for countering Soviet aggression.<sup>55</sup> The new strategy required McNamara to assess and decide what weapon systems and organizational employment would best support the strategy, which called for developing robust capabilities to respond to threats below the nuclear threshold. McNamara believed that the existing decision-making system within the Department of Defense was based on experience and intuition instead of scientific analysis and quantitative estimates.<sup>56</sup> Therefore, McNamara

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<sup>52</sup> Charles R. Shrader, *History of Operations Research in the United States Army*, vol. 2, 1961-1973 (Washington, DC: Government Printing Office, 2008), 38.

<sup>53</sup> Andrew A. Gallo, "Understanding Military Doctrinal Change During Peacetime" (PhD diss., Columbia University, 2018), 149, accessed December 30, 2018, <https://doi.org/10.7916/D8709HB9>.

<sup>54</sup> Carter, *The US Army Before Vietnam, 1953-1965*, 44.

<sup>55</sup> Shrader, *History of Operations Research in the United States Army*, vol. 2, 1961-1973, 38.

<sup>56</sup> *Ibid.*, 39.

expanded the realm of operations research through the ‘whiz kids’ who collectively staffed the Office of Systems Analysis.<sup>57</sup>

The US Army identified the Soviet military as its greatest threat following Vietnam. Concurrently, the 1973 Arab-Israel War illustrated the lethality of modern war. General Creighton Abrams, Chief of Staff of the US Army, identified that the Army needed to be reoriented and retrained to counter the conventional threat of the Soviets and ordered the establishment of Training and Doctrine Command. General William DePuy was appointed the Commander of TRADOC.<sup>58</sup> DePuy acknowledged that war against the Soviet Union meant fighting outnumbered and winning. Under immense budgetary pressure, DePuy utilized a systems-based approach to weapons acquisitions and developed a defensive doctrine that could be executed within the Army’s existing authorized total manpower. The defensive doctrine was termed “Active Defense” and encapsulated in the 1976 version of FM 100-5, *Operations*.<sup>59</sup> The defensive nature of the manual was oriented around the utilization of force ratios to determine the best method of employing units. The essential idea behind the doctrine of active defense is the lateral movement of forces to the point of concentration where the enemy commits their attack. This method accepts risk on the flanks to achieve local numerical superiority at the chosen location of concentration. The result was to cede some ground to wear down the enemy through attrition.<sup>60</sup>

The 1976 version of *Operations* was the first in the series to incorporate force ratios as a decision-making tool. The manual heavily emphasized the favorability of defending with a ratio of 3:1, mentioning it in five varying forms of application at both the tactical and the operational

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<sup>57</sup> Shrader, *History of Operations Research in the United States Army*, vol. 2, 1961-1973, 45.

<sup>58</sup> Richard M. Swain, “Filling the Void: The Operational Art and the U.S. Army,” in *Operational Art: Developments in the Theories of War*, ed. B.J.C. McKercher and Michael Hennessy (Westport, CT: Praeger, 1996), 149.

<sup>59</sup> Gallo, “Understanding Military Doctrinal Change During Peacetime,” 150.

<sup>60</sup> *Ibid.*, 150-151.

levels of war, similar to the argument advanced by Lanchester. First, the manual describes the tactical fight consisting of lower echelons such as the company and states that the advantages of the defense are so great that the defender should be capable of defeating an “attacker that is superior in combat power by a ratio of about 3:1.”<sup>61</sup> Second, from an operational standpoint, it says that commanders must deploy their forces so that on the defense they are never outnumbered or outgunned by more than 3:1 at the point and time of decision.<sup>62</sup> Next, the manual describes concentration and states that “as a rule of thumb, [generals] should not seek to be outweighed more than 3:1 in terms of combat power.”<sup>63</sup> “Operational Art” and the operational level of war did not yet exist in the Army’s lexicon in 1976 and did not enter it until the 1982 edition of *Operations*.<sup>64</sup> However, when the manual describes the role of generals, it is referring to operations that military planners today understand to exist between the operational and tactical levels of war. The manual later describes the advantages of the defender “with full night capability multiplies his weapons effectiveness and, therefore, can defend against combat power ratios which otherwise might be greater than 3:1.”<sup>65</sup> Finally, the manual indicates that in addition to nighttime, a defender can defeat attacking forces with relative combat power greater than 3:1 by defending in urban areas.<sup>66</sup>

The 1976 version of *Operations* approached warfare in a “scientific” manner, for which it received abundant criticism. The manual alarmed traditionalists with its abundant use of graphs and charts, associated with operations research analysts, aimed at illustrating the lethality of the battlefield. Many military professionals associated the manual and its quantitative approach with

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<sup>61</sup> US Department of the Army, *Field Manual (FM) 100-5, Operations* (Washington, DC: Government Printing Office, 1976), 3-4.

<sup>62</sup> *Ibid.*, 3-5.

<sup>63</sup> *Ibid.*, 5-3.

<sup>64</sup> Swain, “Filling the Void: The Operational Art and the U.S. Army,” 159-160.

<sup>65</sup> US Army, *FM 100-5* (1976), 5-10.

<sup>66</sup> *Ibid.*, 14-25.

McNamara's quantitative management of Vietnam, which they blamed for losing the war.<sup>67</sup>

Approaching warfare with a scientific approach, *Operations* uses the 3:1 rule inconsistently, at different levels of war. In particular, it states that generals, through what is today understood as the elements of operational art including basing, tempo, and decisive points seek to achieve a 3:1 advantage against the enemy. Likewise, the manual stated that at the tactical level, commanders utilize advantageous terrain, obstacles, and superior weapons systems to defend against an attacker who is superior in combat ratio by 3:1.<sup>68</sup> Moreover, it is unclear how the users of the manual should compute "combat power." The manual only applies the 3:1 ratio to defensive cases, stating that to overcome defenders, attackers need a ratio of at least 6:1.<sup>69</sup> The manual does not introduce any other force ratios that are captured in current doctrine, noted in Table 1 above.

The Army replaced the doctrine of Active Defense with 'AirLand Battle' in the 1982 version of FM 100-5: *Operations*. The Army viewed DePuy's version of *Operations* as overly simplistic, ignoring the human dimension and ultimately rejected it as "too mechanical, too mathematically certain, too specific."<sup>70</sup> Lieutenant General Donn Starry, US Army's V Corps commander when DePuy's Active Defense doctrine was published, eventually replaced him as the TRADOC commander. Starry realized that the active defense doctrine assumed the Soviet Union would adhere to a doctrine of a massed penetration at a single point. While the V Corps commander in Europe, Starry realized that the Soviet Army modified their doctrine to include multi-pronged attacks across multiple axis of advance. Therefore, Starry focused AirLand Battle on Army and Air Force integration to better strike across both the width and depth of the enemy forces. The new doctrine went beyond just systems and focused on the human dimension and

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<sup>67</sup> Swain, "Filling the Void: The Operational Art and the U.S. Army," 151.

<sup>68</sup> US Army, *FM 100-5* (1976), 3-4.

<sup>69</sup> Ibid.

<sup>70</sup> Paul Herbert, "Deciding What has to be Done – General DePuy and the 1976 Edition of FM 100-5," *Leavenworth Papers* no. 16 (Ft. Leavenworth, KS: Combat Studies Institute, 1988), 101.

psychological impact of such integrated operations.<sup>71</sup> As a result, the emphasis on force ratios within doctrine waned. Scholars, however, continued to argue over force ratios, particularly in articles that appeared in *International Security* debating the effectiveness of various doctrines to defend NATO forces.

## The “Great Debate” of the 1980s

As the Cold War continued, academics pitted DePuy’s ‘Active Defense’ doctrine against ‘AirLand Battle.’ Many renowned scholars joined in the debate, examining the NATO defense of Europe against an offensive incursion of the USSR. It began with DePuy’s 1979 article in *Army* magazine that focused on Active Defense and the suitability of the doctrine as the basis of NATO’s defense plan. Then, in 1982 John Mearsheimer reinforced DePuy’s argument when he defended NATO’s Forward Defense of Europe in an article published in *International Security*. This debate lasted until 1989, with Samuel Huntington, Eliot Cohen, Joshua Epstein, and Trevor Dupuy all submitting contributions to *International Security*. As 1982 was also the same year that AirLand Battle replaced Active Defense in the Army’s FM 100-5: *Operations*, the debate was also about AirLand Battle and Active Defense, with the authors often using the metric of force ratios to build their arguments. Ultimately, the debate failed to develop consensus as to which level of war force ratios applied, and their ability to consistently account for historical cases.

DePuy’s 1979 article argued for superior technology and tactics as the basis of the NATO defense plan. He utilized a similar scientific approach in the 1976 edition of *Operations* and specifically the 3:1 force ratio. DePuy stated that personnel strength ratios did not equate to combat power but was a useful starting point for conducting analysis, and that conventional wisdom generated the 3:1 ratio for the defense and the 6:1 ratio for an attacking force. As evidence of his argument and the general wisdom of the ratios, he cited S. M. Shtemenko, former Soviet Chief of General Staff whose book *Last Six Months* focused on the World War II

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<sup>71</sup> Gallo, “Understanding Military Doctrinal Change During Peacetime,” 167-169.

engagements between Russia and Germany. In addition, DePuy argued that Army Materiel Systems Analysis Agency at Aberdeen Proving Ground, Maryland – employing the HERO database – determined that the actual ratio was 2.6, but the Army went with 3 because it was “a good round figure.”<sup>72</sup> Because the existing ratio in Europe at that time was around 2:1 in favor of the Warsaw Pact forces, there was very little room for error by the NATO commanders, as operationally the attackers could easily convert local advantages of 2:1 to greater than 3:1.<sup>73</sup>

DePuy seemed unaware that the 3:1 force ratio had been used previously within *Maneuver Control*, and that usage may have contributed to the decision to round the number from 2.6 to 3. A larger problem is his interchanging of the levels of warfare to which force ratios are applicable. He argues both at a holistic NATO defense level against the USSR as well as at local or tactical levels. Finally, he compared the relative strength of Soviet and US forces in terms of divisions, individual tanks, and infantry soldiers across the entire frontage, before concentration or reinforcing echelons, when he computed relative combat power expressed in ratios.<sup>74</sup> Thus he further perpetuates the confusion and lack of definition surrounding the utilization of force ratios, specifically the 3:1.

In 1982, John J. Mearsheimer published an article in *International Security* that focused on why the NATO “Forward Defense” plan – by which he meant Active Defense – was sufficient to defeat a Warsaw Pact blitzkrieg attack into Western Europe.<sup>75</sup> Mearsheimer was primarily known amongst scholars and military professionals for his work covering deterrence theory. He compared available manpower and military weapons systems of both sides, concluding that the Warsaw Pact nations could not achieve a force ratio greater than 2:1 at the operational level of

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<sup>72</sup> William DePuy, “Technology and Tactics in Defense of Europe,” *Army* (April 1979): 280.

<sup>73</sup> *Ibid.*

<sup>74</sup> *Ibid.*

<sup>75</sup> John J. Mearsheimer, “Why the Soviets Can't Win Quickly in Central Europe,” *International Security* 7, no. 1 (Summer 1982): 13-14.

war. Citing both Soviet and US Army Doctrine, Mearsheimer claimed that between a 3:1 and 5:1 ratio is required to overwhelm a defense.<sup>76</sup> More specifically, he asserted that the NATO Forward Defense plan would require attacking forces to advance along six projected axes and would only be capable of massing ten divisions along each axes giving them a force ratio of 2.5:1 of armored divisions, which he deemed “hardly satisfactory.”<sup>77</sup> Mearsheimer argued that it is impossible to explain the outcomes of many battles by force ratios alone. However, he believed that when one side has an overwhelming advantage in forces that the asymmetry is likely to lead to a decisive victory, citing the German’s ability to overwhelm the Polish in 1939 as an example.<sup>78</sup> Mearsheimer argued that in a conventional war in Europe, the Warsaw Pact nations would lack numerical superiority, and therefore their success would rely on operational art.<sup>79</sup>

In 1983, Samuel Huntington fielded the first critique of Mearsheimer’s article. Huntington pointed out that the 3:1 ratio is a cliché. He argued that what is important is not a matter of overall superiority in numbers (at what is today considered the operational level of war), but the amount of force required at an exact point of attack determines an outcome. Hence a unit at the tactical level of war may utilize mobility, deception, and surprise to achieve greater than 3:1 at a specific point, and therefore be capable of penetrating a defensive position.<sup>80</sup> Pointing out that Mearsheimer presumed foreknowledge of chosen Soviet avenues of advance, Huntington leveled his criticism at the inability of NATO Forward Defense to offset the Soviet Army’s

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<sup>76</sup> Mearsheimer, “Why the Soviets Can’t Win Quickly in Central Europe,” 15.

<sup>77</sup> Ibid., 16-17.

<sup>78</sup> Ibid., 9-10. Although Mearsheimer cited an overwhelming force ratio leading to a decisive victory, other authors have varying interpretations. Dennis Showalter attributed the German victory in Poland to a multitude of factors, including those that make up the concept of blitzkrieg and the intervention of the Soviet Union. For more information, see Dennis Showalter, “Prussian-German Operational Art, 1740-1943,” in *The Evolution of Operational Art: From Napoleon to Present*, ed. John Olsen and Martin van Creveld (Oxford: Oxford University Press, 2011), 52.

<sup>79</sup> Mearsheimer, “Why the Soviets Can’t Win Quickly in Central Europe,” 9-10.

<sup>80</sup> Samuel P. Huntington, “Conventional Deterrence and Conventional Retaliation in Europe,” *International Security* 8, no. 3 (Winter 1983-1984): 46.



operational advantage in terms of its ability to leverage superiority in mobility, deception, and surprise. He therefore believed that NATO's Forward Defense plan ensured a military defeat. Rather than more forces, Huntington argued that a change in mind-set was required, and believed AirLand Battle met that need.<sup>81</sup> Essentially, Huntington was in favor of elevating the operational art involved in war above the purely scientific approach.

Joshua Epstein was the next to use the pages of *International Security* to criticize Mearsheimer's argument and his utilization of the 3:1 rule. In 1988, Epstein joined in the debate by pointing out that the rule fails to describe what units are to be measured. Moreover, statistical analyses of historical samples of breakthrough operations had failed to validate the 3:1 rule. Epstein utilized databases compiled by Trevor Dupuy and HERO to demonstrate that 3:1 or even 2:1 is not necessary for an attacker to win at the point of attack or, therefore, *tactically*. Epstein used the databases to illustrate that in 50% of the engagements studied, despite being outnumbered, the attacker succeeded in accomplishing its objective. Epstein also noted that the inclusion of the 3:1 rule in US Army doctrine is unsurprising because any military commander would "prefer" more force to less, and that it is simply a point of departure for military leaders to argue in favor of additional resources.<sup>82</sup>

Mearsheimer responded to Epstein's article the following year by narrowing the focus, asserting that the 3:1 rule only applies to breakthrough battles. He argued that Epstein utilized historical examples that included battles other than breakthroughs and therefore are not applicable to the argument and use of the 3:1 rule.<sup>83</sup> In his counterargument, Mearsheimer outlined what he considered both the deductive and empirical basis for the 3:1 rule. Deductively, it stems from the premise that a defender chooses the terrain, fortifies their position, emplaces obstacles, and direct

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<sup>81</sup> Huntington, "Conventional Deterrence and Conventional Retaliation in Europe," 47.

<sup>82</sup> Joshua Epstein, "Dynamic Analysis and the Conventional Balance in Europe," *International Security* 12, no. 4 (Spring 1988): 155-158.

<sup>83</sup> John Mearsheimer, "Assessing the Conventional Balance: The 3:1 Rule and its Critics," *International Security* 13, no. 4 (Spring 1989): 54-55.

their weapons systems in such a manner that they can inflict more casualties than an exposed attacker can inflict on the defense.<sup>84</sup> Therefore, unless the attacker is numerically superior, it must break off the attack due to losses incurred during battle.<sup>85</sup> He also argued that the empirical basis for the rule was grounded in a belief by modern professional armies and it dated back to the Franco-Prussian War.<sup>86</sup> Additionally, he cited DePuy's 1976 version of FM 100-5: *Operations*, which noted it appeared in both Soviet and US doctrine.<sup>87</sup>

This debate demonstrates that throughout the 1980s, there was no consensus on the application of force ratios. Various authors utilized force ratios, specifically the 3:1 rule to bolster their arguments, but each did so differently. Some such as Mearsheimer argued that force ratios in terms of total numerical superiority mattered. Others such as Huntington pointed out that force ratios were a defunct cliché, and that if they mattered at all it was tactically at a point of concentration. Epstein argued that numerical strengths were ill-defined and therefore an invalid basis of comparison. Moreover, their varied uses of historical examples highlighted the database limitations and the ability to restrict or expand a database to produce favorable statistics. With the collapse of the Soviet Union, the debate ended prematurely, leaving unverified, unvalidated force ratios as a legacy in Army doctrine, where they remain today.

## To the 21<sup>st</sup> Century

Efforts to quantitatively describe and analyze warfare have persisted though the 1990s and early 2000s. Christopher Lawrence, for example, worked for Trevor Dupuy early in his career and currently serves as the Director and President of the Dupuy Institute.<sup>88</sup> His book *War by Numbers* is an attempt to utilize the vast amount of data contained in the DuWar databases and

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<sup>84</sup> Mearsheimer, "Assessing the Conventional Balance: The 3:1 Rule and its Critics," 57.

<sup>85</sup> Ibid.

<sup>86</sup> See Footnote 4.

<sup>87</sup> Mearsheimer, "Assessing the Conventional Balance: The 3:1 Rule and its Critics," 59-60.

<sup>88</sup> "About Christopher A. Lawrence," The Dupuy Institute, accessed September 27, 2018, <http://www.dupuyinstitute.org/blog/author/chris/>; Lawrence, *War by Numbers*, xi.

extrapolate out the various factors and level of influence they have on the outcome of engagements.<sup>89</sup> Published in 2017, this is the most contemporary contribution to the literature of force ratios and includes analysis of more recent engagements than were available to Lanchester or Dupuy. Considering fifty-six “verities” of Dupuy's work relating to combat, advance rates, and attrition, the book validated five using historical analysis: “(1) Defensive strength is greater than offensive strength; (2) Surprise substantially enhances combat power; (3) There is no direct relationship between advance rates and force strength ratios, (4) Casualty rates of small forces are higher than those of large forces; and (5) There is no direct relationship between force ratios and casualty rates.”<sup>90</sup>

Lawrence utilized the Tactical Numerical Deterministic Model (TNDM), which succeeded the QJM, to conduct his analysis, and more specifically to determine the winner and loser of an engagement, assess personnel and equipment losses, and determine the rate of advance. It successfully replicated the outcomes of the fifty-two engagements from the 1967 and 1973 Arab-Israeli Wars with a 90 percent accuracy. The TNDM also successfully predicted success/failure in 85 percent of another validation attempt consisting of sixty-six engagements, which included 35 engagements from the earlier analysis. In 1990, the TNDM was used to predict the outcome and casualties of the impending war with Iraq (Operation Desert Storm), proving to be the most accurate estimate of casualties. Since then, the TNDM had undergone three semi-independent validation efforts, one each at corps-level, division-level, and battalion-level operations. The cases represent engagements ranging from World War I, World War II, and post-1945 conflicts including Vietnam, the Arab-Israeli Wars, the Falklands War, and others. The TNDM predicted success correctly in 21 of 24 corps level cases, 24 of 25 division-level cases, and 64 of 76 battalion-level cases.<sup>91</sup>

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<sup>89</sup> Lawrence, *War by Numbers*, 6-7.

<sup>90</sup> *Ibid.*, 325.

<sup>91</sup> *Ibid.*, 299-304, 320-322.

These results support the notion that the outcome of engagements can be systematically and scientifically determined beforehand. In essence, Lawrence's work and the TNDM bolster arguments for a scientific approach to war in lieu of an operational art approach. While TNDM has demonstrated that it can provide a strategic assessment in the form of a probability of success for an overall effort, it has not demonstrated its effectiveness as a tool for operational planners. Among others, its limitations include a lack of considerations for the naval component, air forces beyond the tactical air support level, or any logistical results such as ammunition expenditure. Moreover, maintaining the accuracy of the TNDM requires an enormous amount of inputs.<sup>92</sup> As many of these rely upon difficult-to-obtain information, this requirement incurs a massive amount of lead time. As a result, the TNDM is more frequently used by companies to develop requirements that drive the development of hypothetical weapons more so than operational planners.<sup>93</sup> Moreover, it is ineffective in situations where planners or commanders face a large amount of uncertainty.

In addition to the TNDM, this period also saw other contributions towards developing quantitative deterministic models for predicting the outcomes of battles. The primary one that continues to impact the US Army today is the COFMs. These were first developed by the Soviet Union as a means to mathematically compare the strengths of opposing forces going beyond static force comparisons and accounting for time and temporal characteristics as well.<sup>94</sup> The US borrowed COFMs from the Soviets, and current US versions apply a numerical value to

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<sup>92</sup> "And Now, the War Forecast Software: Can software really predict the outcome of an armed conflict, just as it can predict the course of the weather," *Economist*, September 25<sup>th</sup> 2005, accessed February 7, 2019, <https://www.economist.com/technology-quarterly/2005/09015/and-now-the-war-forecast>. The report highlights that the Dupuy Institute often has to pay clerical works to photocopy technical manuals and often interact with an assortment of contacts within the defense industry for their information.

<sup>93</sup> Ibid.

<sup>94</sup> James Womack, "Soviet Correlation of Forces and Means: Quantifying Modern Operations" (master's thesis, US Army Command and General Staff College, 1990), 3.

intangible factors, such as morale, training, terrain, weather, and leadership.<sup>95</sup> To communicate the chance of success, the COFM currently used by TRADOC Analysis Center displays the correlation in the terms of a force ratio. In addition to expressing the corresponding force ratio, the tool estimates the number of casualties that each force should expect.<sup>96</sup>

The main problems with utilizing COFMs are that the force ratios it uses to communicate chances of success have not been historically validated (as noted earlier), and it may impede commanders' initiative and planning, as Omar Bradley suggested years ago. Another problem with COFMs is that, like force ratios, no consensus exists on definitions and their usage, which degrades their possible utility. For example, current COFMs, and force ratios in general, apply a combat power potential to organizations based on the capabilities of a weapons system, typically derived from its range and rate of fire just like Lanchester's model. However, many weapons that the US Army employs are situational weapons such as the shoulder fired missiles of an Anti-Tank Platoon. Neither COFMs, nor force ratios, account for the difference in the combat power potential that this platoon would have against a mechanized formation, as opposed to fighting against a light infantry organization.<sup>97</sup> Multiple versions of correlation of forces calculators exist which introduces confusion more than clarity. In 1992, David Hogg outlined the four various correlation of forces tools employed throughout the Army and he highlighted the variance in databases and assumptions behind each. Throughout his study, he identified the lack of consistency and cautioned that the system led to each unit determining correlation of forces differently, threatening the legitimacy of training and posing potentially disastrous consequences

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<sup>95</sup> Dale Spurlin and Matthew Green, "Demystifying the Correlation of Forces Calculator," *Infantry* (January-March 2017): 14.

<sup>96</sup> Michael Murry, post to "Correlation of Forces (CoFMs)," January 22, 2019 (7:14 a.m. CST), *Correlation\_of\_Forces\_Calculator\_Version\_2017.01, Microsoft Excel Worksheet*, accessed February 2, 2019, <https://www.milsuite.mil/book/docs/DOC-569300>.

<sup>97</sup> Spurlin and Green, "Demystifying the Correlation of Forces Calculator," 16-17.

in war.<sup>98</sup> Lastly, like the TNDM, the accuracy of COFMs depends on a near perfect knowledge of enemy forces to include subjective factors such as morale, discipline, and training proficiency.<sup>99</sup> In future conflicts where planners deal in conditions surrounded by uncertainty, the accuracy of tools such as TNDM and CoFMs is reduced. Hogg recommended that to mitigate some of these problems, TRADOC and specifically Fort Leavenworth needed to assume the role as lead agency to standardize a methodology for computing a correlation of forces, and it should consider the use of the QJM as a baseline as it was the most realistic of the four models available then.<sup>100</sup>

Unfortunately, the Army has not heeded Hogg's advice. As of 2017, there were still multiple versions of correlation of forces tools available to planners.<sup>101</sup> Compounding the problem of differing models is that the underlying data used for each is not apparent to the user, who remains unaware as to the context and historical information used to develop the combat value of systems or units. Therefore, there is a potential that a correlation of forces model developed during the height of counter-insurgency operations in Baghdad or Mosul, for example, would provide a relatively low effectiveness of the Army's main battle tank. Without knowing the data that was used to derive combat values, military planners may utilize such a tool when trying to determine the relative combat power in a large-scale combat operation in which main battle tanks are well suited. Correlation of forces tools must be centrally controlled, transparent, and undergo verification, validation, and accreditation if they are to be of use to military planners. Even then, extreme caution must accompany the use of a correlation of forces tool as the time spent utilizing a tool such as this is time not spent on critical thinking and forming an individual assessment of the problem.

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<sup>98</sup> Hogg, "Correlation of Forces: The Quest for a Standardized Model," 39.

<sup>99</sup> Womack, "Soviet Correlation of Forces and Means: Quantifying Modern Operations," 96.

<sup>100</sup> Hogg, "Correlation of Forces: The Quest for a Standardized Model," 40.

<sup>101</sup> Spurlin and Green, "Demystifying the Correlation of Forces Calculator," 14.

## Current Manifestations

Paul Herbert considers doctrine as a point of departure for the Army's performance in war, and as such it has a profound consequences for its effectiveness.<sup>102</sup> He continues by highlighting that the Army's doctrine is a product of the Army's "past, present, and vision of its future."<sup>103</sup> Today force ratios, and particularly the 3:1 ratio, have become codified within Army doctrine and culture, even though the scholars have never reached a consensus on their use. In October 2017, the US Army published its most recent edition of FM 3-0: *Operations*. The manual stresses the need for the US Army to adapt and prepare for LSCO, stressing the need for realistic and repetitive training.<sup>104</sup> The manual continues to perpetuate the use of force ratios as a planning heuristic by incorporating them despite a lack of understanding of their intended application.<sup>105</sup> Similarly, modified variations of the Lanchester equations continue to form the basis of analysis in many of the Army's combat simulations.<sup>106</sup>

Force ratios also remain prevalent to debates at the strategic level. During Vietnam, it was argued that a 10:1 force ratio was required for counterinsurgencies. The 10:1 ratio again resurfaced in 2007 in the Army's counterinsurgency doctrine heralded by General David Petraeus. However, like conventional force ratios, the 10:1 counterinsurgency ratio has not been historically grounded and emphasizes the science of war. Instead of a correlation of manpower,

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<sup>102</sup> Paul H. Herbert, "Toward the Best Available Thought: The writing of Field Manual 100-5, "Operations" by the United States Army, 1973-1976" (PhD diss., Ohio State University, 1985), 2.

<sup>103</sup> *Ibid.*, 6.

<sup>104</sup> US Department of the Army, *Field Manual (FM) 3-0, Operations* (Washington, DC: Government Printing Office, 2017), foreword.

<sup>105</sup> *Ibid.*, 7-28.

<sup>106</sup> James Zanella, "Combat Power Analysis is Combat Power Density" (monograph, School of Advanced Military Studies, US Army Command and General Staff College 2012), 11.

whether counting individuals or battalion equivalents, intangible factors such as social issues are more important for defeating an insurgency.<sup>107</sup>

Other organizations beyond the Army have also continued to explore the utility of force ratios. RAND Corporation utilized a modified version of Lanchester's equations in 2016 during a wargaming study that examined the probable outcome in the event of a Russian invasion into the Baltic nations of Estonia, Latvia, and Lithuania.<sup>108</sup> Force ratios appear to be leaping forward into the future, too. In 2018 at the Association of the United States Army conference, the idea that artificial intelligence would soon be capable of assessing the force ratios for planners was advanced.<sup>109</sup> These current manifestations illustrate the depth and breadth which force ratios have become ingrained into the US Army's way of thinking.

## Conclusion

Acknowledging the role that chance plays in war, Clausewitz stated that "so-called mathematical, factors never find a firm basis in military calculations."<sup>110</sup> As the modern Army struggles to transition from counter-insurgency to large scale combat operations, it should encourage a debate over its practices, specifically the heuristic approaches that military planners rely on to save time during planning. Despite flaws, the prevalence of force ratios within Army doctrine and culture remains. Force ratios are a derivative of Lanchester's early work on

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<sup>107</sup> Joshua Thiel, "COIN Manpower Ratios: Debunking the 10 to 1 Ratio and Surges," *Small Wars Journal* (January 2011), accessed January 1, 2019, <http://smallwarsjournal.com/jrnl/art/coin-manpower-ratios-debunking-the-10-to-1-ratio-and-surges>.

<sup>108</sup> David A. Shlapak, and Michael W. Johnson, *Reinforcing Deterrence on NATO's Eastern Flank* (Santa Monica, CA: RAND Corporation, 2016), accessed February 2, 2019, [https://www.rand.org/pubs/research\\_reports/RR1253.html](https://www.rand.org/pubs/research_reports/RR1253.html).

<sup>109</sup> James Dubik, "Decision-Making Process May Need Update," Association of the United States Army, November 7, 2018, accessed December 30, 2018, <https://www.ausa.org/articles/decision-making-process-may-need-update>.

<sup>110</sup> Carl von Clausewitz, *On War*, ed. by Michael Howard and Peter Paret (Princeton: Princeton University Press, 1976), 86.



concentration and attrition but do not account for technological developments and the multiple domains of warfare that make up the modern battlefield.<sup>111</sup>

The Army must differentiate force ratios from correlation of forces models. Force ratios should be abandoned as invalid heuristics. Correlation of forces models, with some effort, may provide utility to planners if they can be separated from force ratios and altered to present the results of its comparison in terms of anticipated effects and expenditures. By continuing to present results in the form of a force ratio that is not valid to begin with, the tool will lack utility. If altered, COFMs could be used to identify likely casualty numbers and expenditure rates for ammunition that would be required to achieve a desired effect (both the QJM and the TNDM support such applications).

As Kahneman cautioned, relying on heuristics may lead to prediction errors. Bradley observed this danger, observing that force ratios led commanders to constrain their options when assessing the battlefield. For this and other reasons previously identified, the US Army should abandon force ratios as a planning heuristic. Planners should focus on operational art and achieving surprise to give tactical commanders the best chance at success.

As the Army continues to expand simulations as cost-effective means of training, it should cultivate further debate into the quantitative analysis that they are built on. At a minimum, a renewed debate around force ratios could result in updating, centralizing, and publishing the analysis that goes into calculators such as the correlation of forces models. The debate could also settle on definitions and explanations, informing leaders at all echelons so that they will be better prepared for large-scale combat operations.

Finally, and not addressed previously in this monograph, is the opportunity that the study of force ratios affords the military. Studying other nations' development and adherence to force

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<sup>111</sup> Schneider, "Vulcan's Anvil: The American Civil War and the Foundations of Operational Art," 9.

ratios, such as Russian doctrine, may provide an advantage in the event of any future conflict. Just as understanding the bias within our own way of thinking and adherence to force ratios is a risk, understanding an adversary's quantitative or scientific approach to warfare may provide an opportunity.

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